

# $H_\infty$ Active Suspension Control Based on Moving Horizon Strategy\*

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**Abstract-** A moving horizon  $H_\infty$  disturbance attenuation scheme for active suspensions with time-domain constraints is suggested, where closed-loop dissipation and hence  $H_\infty$  performance are guaranteed. By minimizing the performance level on-line, the closed-loop system is able to relax the performance requirement so as to meet hard constraints in the case of unforeseen large disturbances, and to recover a better performance in the case of generally mild disturbances. Simulation results for a 2 DOF quarter-car model are presented.

**Index Terms-** Active suspensions, time-domain constraints,  $H_\infty$  control, moving horizon strategy, dissipation.

## I. INTRODUCTION

Performance requirements for advanced vehicle suspensions include isolating passengers from vibration and shock arising from road roughness (ride comfort), suppressing the wheels so as to maintain firm, uninterrupted contact of wheels to road (good handling or good road holding) and keeping suspension strokes within an allowable maximum (cf.[1]). In order to manage the trade-off between these conflicting requirements, many active suspension control approaches are proposed (e.g. [2], [3], [4], [5], [6], [7], [8] and the references therein), based on various control techniques such as LQG, adaptive control and nonlinear control. Recently,  $H_\infty$  active suspensions are intensively discussed in the context of robustness and disturbance attenuation (e.g. [9], [10], [11], [12], [13], [14] and the references therein).

Since requirements for good handling and keeping suspension strokes within an allowable maximum are in fact time-domain constraints, the active suspension control problem can be considered as a disturbance attenuation problem with hard constraints [15]. Hence, [16] suggests an LMI based constrained  $H_\infty$  control approach to design active suspensions, where hard constraints are handled in their nature form and separately from the specification of ride comfort. However, in order to be prepared for unforeseen large disturbances, arising from for example a pronounced bump or pothole on an otherwise smooth road, one should choose larger values of the controller parameter  $\alpha$ , which leads to worse ride comfort (larger performance level  $\gamma$ ) even if the actual disturbance is rather mild. On the other hand, enforcing

better ride comfort (smaller  $\gamma$ ) requires smaller values of  $\alpha$ , which might result in constraint violations in case that the system is affected by unexpectedly large disturbances. These cases can be indicated in the bump responses of the constrained  $H_\infty$  active suspension [16]. In order to overcome this dilemma, this paper continues the work of [16] along the line of on-line manage the satisfaction of constraints and the level of performance by implementing the constrained  $H_\infty$  control scheme in a moving horizon manner, that is well-known in the literature of model predictive control.

The paper is organized as follows: Section II summarizes firstly the LMI based constrained  $H_\infty$  disturbance attenuation scheme discussed in [16]. Then, we implement this control scheme in the moving horizon fashion and discuss the stability and performance properties of the closed-loop system. In Section III, we apply the moving horizon  $H_\infty$  disturbance attenuation scheme to active suspension control, in the basis of a 2 DOF quarter-car model.

## II. MOVING HORIZON $H_\infty$ CONTROL SCHEME

### A. LMI based constrained $H_\infty$ disturbance attenuation [16]

For a generality, we consider the following system

$$\dot{x}(t) = Ax(t) + B_1w(t) + Bu(t) \quad (1a)$$

$$z_1(t) = C_1x(t) + D_1w(t) + D_{1u}u(t) \quad (1b)$$

$$z_2(t) = C_2x(t) + D_{2u}u(t) \quad (1c)$$

subject to output and control constraints

$$|z_{2i}(t)| \leq z_{2i,\max}, i = 1, 2, \dots, p_2, t \geq 0, \quad (2a)$$

$$|u_i(t)| \leq u_{i,\max}, i = 1, 2, \dots, m_2, t \geq 0, \quad (2b)$$

Here  $x \in \mathbb{R}^n$  is the vector of states,  $w \in \mathbb{R}^m$  is the vector of disturbance inputs,  $u \in \mathbb{R}^{m_2}$  is the vector of control inputs,  $z \in \mathbb{R}^{p_1}$  is the vector of  $H_\infty$  performance controlled outputs and  $z \in \mathbb{R}^{p_2}$  is the vector of constrained outputs. A fundamental assumption for the system (1) is that  $(A, B)$  is stabilizable and  $(C_1, A)$  is detectable. Our control problem is to design a controller such that the closed-loop system is inter-

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nally stable, and the  $H_\infty$  norm from the disturbance  $w$  to the performance output  $z_1$  is minimized, while the control and output constraints are respected.

We consider the state feedback case with  $u = Kx$  and provide an LMI optimization based solution as follows: Suppose that for a given  $\alpha > 0$ , the semi-definite programming

$$\min_{\gamma, Q=Q^T > 0, Y, X, Z} \gamma \quad (3)$$

Subject to

$$\begin{bmatrix} AQ + QA^T + BY + Y^T B^T & * & * \\ B_1^T & -\gamma I & * \\ C_1 Q + D_{1u} Y & D_1 & -\gamma I \end{bmatrix} < 0 \quad (4a)$$

$$\begin{bmatrix} \frac{1}{\alpha} X & Y \\ Y^T & Q \end{bmatrix} \geq 0, \text{ with } X = X^T, X_{ii} \leq u_{i, \max}^2 \quad (4b)$$

$$\begin{bmatrix} \frac{1}{\alpha} Z & * \\ (C_2 Q + D_{2u} Y)^T & Q \end{bmatrix} \geq 0, \text{ with } Z = Z^T, Z_{ii} \leq z_{2i, \max}^2 \quad (4c)$$

admits an (almost) optimal solution  $(\hat{\gamma}, \hat{Q}, \hat{Y})$ , then, the state feedback  $\hat{K} = \hat{Y}\hat{Q}^{-1}$  guarantees

- (i) a disturbance attenuation level  $\hat{\gamma}$  for all energy bounded disturbances;
- (ii) the  $H_\infty$  norm from the disturbance  $w$  to the performance output  $z_1$  is guaranteed less than  $\hat{\gamma}$ ;
- (iii) the output energy is bounded by  $\hat{\gamma}\alpha$ , if the disturbance energy  $w_{\max}$  and the initial state  $x(0)$  satisfy  $\hat{\gamma}w_{\max} + V(x(0)) \leq \alpha$ ;
- (iv) the hard constraints in (2) are respected, if the disturbance energy  $w_{\max}$  and the initial state  $x(0)$  satisfy  $\hat{\gamma}w_{\max} + V(x(0)) \leq \alpha$ .

In (4), \* represents the transpose of the corresponding element. With a Lyapunov-type function  $V(x) := x^T P x$  as well as  $P = Q^{-1}$ , the feasibility of (4a) leads to the dissipation inequality

$$V(x(t_2)) + \gamma^{-1} \int_{t_1}^{t_2} \|z_1(t)\|^2 dt \leq \gamma \int_{t_1}^{t_2} \|w(t)\|^2 dt + V(x(t_1)) \quad (5)$$

for the system controlled with the state feedback gain  $K = YQ^{-1}$  and for all  $t_2 > t_1 \geq 0$ . Hence, the closed-loop system has a disturbance attenuation level  $\gamma$  for all energy bounded disturbances. Let  $x(0) = 0$  and by  $V(x) \geq 0$ , (5) becomes

$$\int_0^\infty \|z_1(t)\|^2 dt \leq \gamma^2 \int_0^\infty \|w(t)\|^2 dt,$$

which implies the  $H_\infty$  norm from the disturbance  $w$  to the performance output  $z_1$  less than  $\gamma$ .

Moreover, it can be shown from (5) (see [17], [16]) that all perturbed state trajectories stay in an ellipsoid defined by

$$\Omega_1(P, \alpha) := \{x \in \mathbb{R}^n \mid V(x) \leq \alpha\}, \quad (6)$$

if the initial state  $x(0)$  belongs to an ellipsoid defined by

$$\Omega_2(P, \alpha, w_{\max}) := \{x \in \mathbb{R}^n \mid \gamma w_{\max} + V(x) \leq \alpha\} \quad (7)$$

and the disturbance energy is bounded as  $\int_0^\infty \|w(t)\|^2 dt \leq w_{\max}$ .

As a consequence, we infer

$$\begin{aligned} \max_{t \geq 0} |u_i(t)|^2 &= \max_{t \geq 0} |(YQ^{-1})_i x(t)|^2 \leq \max_{x \in \Omega_1} |(YQ^{-1})_i x|^2 \\ &\leq \alpha \left\| \left( YQ^{-\frac{1}{2}} \right)_i \right\|_2^2 = \alpha (YQY^T)_{ii}. \end{aligned} \quad (8)$$

Hence, the feasibility of the matrix inequality (4b) and similarly the feasibility of the matrix inequality (4c) guarantee the satisfaction of constraints on controls and outputs, respectively.

Furthermore, the property (iii) follows from (5), too. And it suggests smaller values of the controller parameter  $\alpha$  for better performance. However, the property (iv) reveals that the smaller the  $\alpha$ , the smaller the disturbance energy allowed for guaranteeing the satisfaction of the time-domain constraints. This motivates to exploit the moving horizon strategy for on-line managing the trade-off between satisfying constraints and achieving high performance.

#### B. Moving horizon implementation

The basis of the moving horizon strategy in model predictive control is solving an optimal control problem on-line at each sampling time  $t_k$ , updated by the actual state  $x(t_k)$  [18].

In the implementation of the above LMI based  $H_\infty$  control scheme in the moving horizon fashion, first of all, we have to include the matrix inequality

$$\begin{bmatrix} \alpha - \gamma w_{\max} & x(t_k)^T \\ x(t_k) & Q \end{bmatrix} \geq 0 \quad (9)$$

into the optimization problem (3) to enforce the actual state  $x(t_k)$  belonging to the ellipsoid (7). For a given  $w_{\max}$ , (9) is an LMI, too. Secondly, a dissipation constraint in the form of

$$\begin{bmatrix} p_0 - p_{k-1} + x(t_k)^T P_{k-1} x(t_k) & x(t_k)^T \\ x(t_k) & Q \end{bmatrix} \geq 0 \quad (10)$$

is required to guarantee the closed-loop moving horizon system dissipative, where  $p_k$  is recursively updated as

$$P_k := P_{k-1} - \left( x(t_k)^T P_{k-1} x(t_k) - x(t_k)^T P_k x(t_k) \right). \quad (11)$$

A more detailed derivation of the dissipation constraint can be found in [17]. This way, the LMI optimization problem (3) becomes

$$\min_{\gamma, Q^T > 0, Y, X, Z} \gamma \text{ subject to LMIs (4), (9) and (10)}. \quad (12)$$

According to the moving horizon principle of model predictive control, the LMI optimization problem (12) will be solved at each sampling time  $t_k$ , updated with the actual state  $x(t_k)$ . This is implementable, since  $P_{k-1}$  and  $P_{k-1}$  have been determined at the previous sampling time  $t_{k-1}$  and are held fixed. If (12) admits an (almost) optimal solution  $(\gamma_k, Q_k, Y_k)$ , we can then define a piece-wise continuous feedback control law as

$$u(t) = K_k x(t), \quad t \in [t_k, t_{k+1}) \quad (13)$$

with  $K_k = Y_k Q_k^{-1}$ . For the closed-loop moving horizon system, we state the following result.

*Theorem 1:* Suppose that

- $(A, B)$  is stabilizable and  $(C_1, A)$  is detectable;
- at each sampling time  $t_k$ , there exist  $\alpha$  and  $w_{\max}$  such that the LMI optimization problem (12) with the actual state  $x(t_k)$  as initial condition admits an (almost) optimal solution.

Then, the closed-loop system with the moving horizon control law (13) has the following properties:

- (a) for vanishing disturbances it is asymptotically stable;
- (b) the time-domain constraints (2) are respected, if the disturbance during each sampling period satisfies

$$\int_{t_k}^{t_{k+1}} \|w(t)\|^2 dt \leq \frac{\alpha_k - x(t_k)^T P_k x(t_k)}{\gamma_k}; \quad (14)$$

- (c) the dissipation inequality

$$\int_0^k \gamma^{-1} \|z_1(t)\|^2 - \gamma \|w(t)\|^2 dt \leq x(t_0)^T P_0 x(t_0) \quad (15)$$

is guaranteed for  $t_k \geq t_0$ , with  $\gamma := \max\{\gamma_0, \gamma_1, \dots, \gamma_k\}$ ;

- (d) the  $H_\infty$  norm from the disturbance  $w$  to the performance output  $z_1$  is guaranteed less than  $\gamma$ .

*Proof:* By the Schur complement, (10) is equivalent to

$$P_0 - P_{k-1} + x(t_k)^T P_{k-1} x(t_k) - x(t_k)^T P_k x(t_k) \geq 0. \quad (16)$$

Substituting (11) into the above matrix inequality recursively, we obtain that the dissipation constraint enforces

$$\sum_{i=1}^k \left( x(t_i)^T P_{i-1} x(t_i) - x(t_i)^T P_i x(t_i) \right) \geq 0. \quad (17)$$

The feasibility of (4a) at the sampling time  $t_k$  implies that the dissipation inequality (5) is satisfied with  $V(x) = x^T P_k x$  and  $\gamma = \gamma_k$ , *i.e.*

$$\begin{aligned} V(x(t_{k+1})) + \int_k^{k+1} \gamma_k^{-1} \|z_1(t)\|^2 dt \\ \leq \int_k^{k+1} \gamma_k \|w(t)\|^2 dt + V(x(t_k)). \end{aligned} \quad (18)$$

Hence, exploiting (18) for  $t_k = t_0, t_1, \dots, t_l$  leads to

$$\begin{aligned} \int_0^{t_l} \gamma(t)^{-1} \|z_1(t)\|^2 - \gamma(t) \|w(t)\|^2 dt \\ \leq x(t_0)^T P_0 x(t_0) - x(t_{l+1})^T P_l x(t_{l+1}) \\ - \sum_{k=1}^l \left( x(t_k)^T P_{k-1} x(t_k) - x(t_k)^T P_k x(t_k) \right), \end{aligned}$$

where  $\gamma(t) = \gamma_k$  for  $t \in [t_k, t_{k+1})$ . By (17), the above inequality becomes

$$\begin{aligned} \int_0^{t_l} \gamma(t)^{-1} \|z_1(t)\|^2 - \gamma(t) \|w(t)\|^2 dt \\ \leq x(t_0)^T P_0 x(t_0) - x(t_{l+1})^T P_l x(t_{l+1}) \end{aligned} \quad (19)$$

with  $\gamma := \max_{t \in [t_0, t_{l+1})} \gamma(t)$ . Moreover,  $\gamma < \infty$  by the feasibility assumption at each sampling time. Duo to  $P > 0$ , we have the property (c) and hence the property (d) for  $x(t_0) = 0$ . Furthermore, the stability property (a) is shown by  $\int_0^\infty \|z_1(t)\|^2 dt < \infty$  and the detectability of  $(C_1, A)$ . At each time  $t_k$ , matrix inequality (9) forces the actual state  $x(t_k)$  belonging to the initial ellipsoid  $\Omega_2(P_k, \alpha_k, w_{\max})$  that in turn guarantees  $x(t) \in \Omega_1(P_k, \alpha_k)$  for any  $t \in [t_k, t_{k+1}]$ , if the disturbance satisfies (14). Hence, we infer

$$\begin{aligned} \max_{t \in [t_k, t_{k+1})} |u_i(t)|^2 &= \max_{t \in [t_k, t_{k+1})} \left| (YQ^{-1})_i x(t) \right|^2 \\ &\leq \max_{x \in \Omega_1} \left| (YQ^{-1})_i x \right|^2 \leq \alpha (YQY^T)_{ii}. \end{aligned} \quad (20)$$

By exploiting the feasibility of (4b), the above inequality becomes

$$\max_{t \in [t_k, t_{k+1})} |u_i(t)| \leq u_{i, \max}.$$

Note that the maximization in (20) is just over one sampling time, that is weaker than (8). Due to the moving horizon implementation, the above inequality is then valid for any  $t_k \geq t_0$ , that implies that the feasibility of (4b) guarantees the satisfaction of control constraints in (2), as required in (b). Similarly, we can show that the feasibility of (4c) guarantees the satisfaction of output constraints in (2).

*Remark 1:* Different from standard MPC schemes, the suggested moving horizon  $H_\infty$  control scheme is based on continuous models and feedback prediction. The solution of the LMI optimization problem (12) at each sampling time constructs a feedback control for the continuous system, and provides a possibility to choose a larger sampling period. This is attractive for practical applications.

### III. APPLICATION TO ACTIVE SUSPENSION CONTROL

We apply in the following the suggested moving horizon  $H_\infty$  control approach to active suspension control, based on a 2 DOF quarter-car model. Fig.1 shows the 2 DOF quarter-car model, where  $(k_s, c_s)$  consist of the so-called passive suspension;  $k_u$  stands for the tire stiffness;  $m_s$  and  $m_u$  represent sprung and unsprung masses, respectively. Moreover,  $x_s - x_u$  is the suspension stroke,  $x_u - x_o$  the tire deflection and  $x_o$  the vertical ground displacement caused by road unevenness;  $u_f$  is the scalar active force generated by a hydraulic actuator.

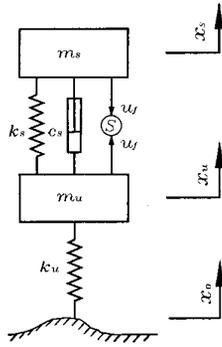


Fig.1. 2 DOF quarter-car model with an active suspension

With a set of state variables  $x_1 = x_s - x_u$ ,  $x_2 = \dot{x}_s$ ,  $x_3 = x_u - x_o$  and  $x_4 = \dot{x}_u$ , the idea dynamics of the quarter-car model can be described by

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & 0 & \frac{c_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{c_s}{m_u} & -\frac{k_u}{m_u} & -\frac{c_s}{m_u} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} w(t) + \begin{bmatrix} 0 \\ \frac{u_s}{m_s} \\ 0 \\ -\frac{u_s}{m_u} \end{bmatrix} u(t),$$

where the normalized active force  $u = \frac{u_f}{u_s}$  is considered as control input and the ground velocity  $w = \dot{x}_o$  represents the disturbance caused by road roughness.

To quantify ride comfort, the body acceleration is in general chosen as performance output, *i.e.*,  $z_1 = \ddot{x}_s$ . In order to ensure a firm uninterrupted contact of wheels to road, the dynamic tire load cannot exceed the static ones [2], *i.e.*,

$$k(x_u(t) - x_o(t)) < (m_s + m_u)g, \forall t \geq 0 \quad (21)$$

Moreover, the suspension stroke limitation in the form of

$$|x_s(t) - x_u(t)| \leq SS, \forall t \geq 0 \quad (22)$$

has to be taken into account to prevent excessive suspension bottoming, which can lead to rapid deterioration of ride comfort and possible structural damage. Both (21) and (22) are time-domain hard constraints. Hence, we choose suspension stroke and relative dynamic tire load as constrained outputs,

*i.e.*  $z_2(t) = \begin{bmatrix} x_s - x_u & \frac{k_u(x_u - x_o)}{(m_s + m_u)g} \end{bmatrix}^T$ . Due to actuator saturation,

it is in addition assumed that the normalized active force is bounded as

$$|u(t)| \leq 1, \forall t \geq 0. \quad (23)$$

As an example, model parameters take the following nominal values (cf.[2]):

$$m_s = 320\text{kg}, k_s = 18 \frac{\text{kN}}{\text{m}}, c_s = 1 \frac{\text{kN} \cdot \text{s}}{\text{m}}, m_u = 40\text{kg},$$

$$k_u = 200 \frac{\text{kN}}{\text{m}}, SS = 0.08\text{m}, u_s = 1.5\text{kN}.$$

The bounds are then  $u_{\max} = 1$ ,  $z_{21, \max} = SS$  and  $z_{22, \max} = 1$ , respectively. We choose  $\alpha = 0.03$  and  $w_{\max} = 0$ , a detailed discussion on the choice of the controller parameter can be found in [16]. Fig.2 shows bump responses, where the corresponding ground displacement is given by

$$x_o(t) = \begin{cases} \frac{A}{2} \left( 1 - \cos\left(\frac{2\pi V}{L} t\right) \right), & 0 \leq t \leq \frac{L}{V} \\ 0, & t > \frac{L}{V} \end{cases}$$

with  $V$  being the vehicle forward velocity,  $A$  and  $L$  being the height and the length of the bump, respectively. The first bump corresponds to  $V = 65 \frac{\text{km}}{\text{h}}$ ,  $L = 5\text{m}$  and  $A = 0.1\text{m}$ , while the second bump starts at the  $t = 1.1\text{s}$  and with  $A = 0.05\text{m}$ . By Remark 1, the sampling period is chosen as  $0.035\text{s}$  that is large compared to the duration of the disturbances ( $0.2769\text{s}$ ). As a comparison, the bump response for the constrained  $H_\infty$  active suspension designed with

$\alpha = 0.03$  is plotted in Fig.2, too. For  $x(0) = 0$ , this fixed  $H_\infty$  controller is guaranteed to respect hard constraints, if the disturbance energy is bounded by  $3.4 \times 10^{-3} \frac{m^2}{s}$  (computed by  $\frac{\alpha}{\gamma}$ ). Since the disturbance energy of the first and the second bump is respectively of  $0.1728 \frac{m^2}{s}$  and  $0.0446 \frac{m^2}{s}$ , the fixed  $H_\infty$  active suspension has no longer guarantee. It can be clearly seen from the bottom picture of Fig.2 that the moving horizon  $H_\infty$  active suspension respects time-domain constraints by relaxing on-line the performance level  $\gamma$ , and recovers the performance level of the fixed  $H_\infty$  active suspension when the extreme strong disturbance (e.g. the first bump) vanishes.

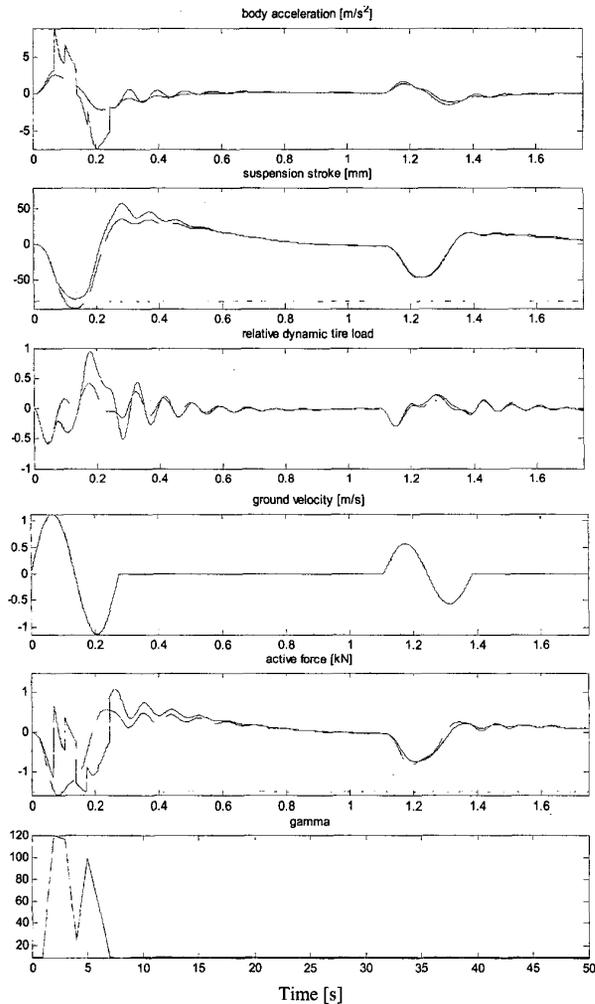


Fig.2. Bump responses: moving horizon  $H_\infty$  suspension (—), fixed constrained  $H_\infty$  suspension B (---)

#### IV. CONCLUSIONS

Active suspension control involves in fact a constrained disturbance attenuation problem, where unexpected large disturbances may be caused by a pronounced bump or pothole.

In order to relax the performance requirement so as to meet time-domain constraints when unforeseen large disturbances happen and to recover a better attenuation level for generally mild disturbances, this paper has suggested a moving horizon  $H_\infty$  control scheme. The solution of the corresponding LMI optimization problem at each sampling time constructs a feedback control for continuous systems. Hence, one can choose a larger sampling period to update the optimization problem, which is attractive for practical applications. Closed-loop dissipation and  $H_\infty$  performance are guaranteed by an additional dissipation constraint.

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